Understanding and Expressing Measurement Uncertainty associated with DC and Low Frequency Metrology

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Introduction

Periodic calibration of electronic systems is required due to the large number and wide variety of components in basic measurement and sourcing systems. In metrology today a calibration laboratory needs to understand how to quantify all the influences and components of measurement uncertainty as they pertain to the laboratory's environment. These uncertainty influences and components are as varied as the cables used in a measurement system to the training level of the technicians tasked with operating this system. Consistent results and confidence in the reported values of a measurement can be achieved with due consideration to all the contributors of uncertainty. This paper is intended to help the laboratory professional begin evaluation of the possible sources of uncertainty and how to formulate a measurement uncertainty budget.

Common terms:

Accuracy is a qualitative expression of the closeness of a measurement's results to the true value.

Precision is a measure of repeatability. A high precision indicates the ability to repeat measurements within narrow limits.

Resolution is the smallest change that can be detected. Generally today with modern instruments this is the smallest increment that can be displayed or LSD.

Uncertainty is a quantitative term that represents a range of values wherein the true value may lie. Uncertainty and confidence is determined using statistical techniques.

Traceability is the ability to relate individual measurement results to national standards or nationally accepted measurement systems through an unbroken chain of comparisons.

Requirements for sound analysis:

Stable Environment; before a laboratory can begin to evaluate the components of uncertainty in a measurement or calibration system, data must be collected to determine if the system is stable. This data may be as simple as monitoring the ambient environment using a temperature and RH logger or as complex as repetitive measurement schemes of the equipment under evaluation.

Proper Training of Personnel; all personnel tasked with performing measurements to assimilation of collected data should be properly trained and evaluated on their understanding of the tasks assigned to them.

Traceable Standards; all standards used in an uncertainty-testing scheme must be traceable for the results to be meaningful.

Uncertainty considerations; all possible sources of uncertainty should be considered from AC line voltage fluctuations to the resolution of the measurement system. A source of uncertainty such as cable EMF may be discarded after determining that the uncertainty is insignificant. It may be appropriate in some test schemes to combine all of the insignificant uncertainties and create a label for this combined uncertainty. Instrument specifications are the most common source of uncertainty data, however proper consideration must be given to the manufacturer's stated confidence level. If the manufacturer did not specify a confidence level, then a rectangular distribution should be assumed, more on distributions later.

Sources of Uncertainty

Uncertainty in the results of a measurement can be affected by many factors, some considerations:

Reference standards and measurement equipment: Uncertainty in their calibration; long term drift; resolution; vibration; electro magnetic interference; sensitivity to change during transportation and handling.

Measurement Setup: cables; shielding; warm up time; thermal voltage influences; measurement probes.

Measurement Process: duration of the measurement; number of measurements; conditioning of standards.

Environmental Conditions: temperature; temperature oscillations; humidity; electromagnetic influences; transients in power source.

Measurement errors

A measurement is subject to many sources of error, some of which can cause an over or under statement of the measurement quantity. While the goal of any metrology lab is to keep these errors small, they cannot be reduced to zero. The challenge for any metrology lab is to find out the quantity of these errors and how large they may be. Measurements are affected by three types of errors; Random, Systematic, and Gross.

Random errors are due to unknown causes and are only detectable when repeated measurements are made with a stable measurement setup and consistent measurement technique. This type of error will result in readings that, when repeated, are not always the same. If the reason for the variation is not obvious, then it falls into the category of a random error. *Note: Random errors cannot be quantified without a stable environment and consistent measurement technique.*

Systematic errors relate to the equipment used in the measurement process or external influences on the equipment. Examples include: loading effects, thermals, drift-rate, leakage currents, and external noise.

Gross errors are caused by the technician and can be strictly controlled with proper training. Examples include: misreading of instrument results, incorrect adjustments, using the wrong instrument, errors in recording calibration data, and computational errors. All of these errors can be avoided with proper training and attention to detail.

Classifications of uncertainty

Type "A" evaluation method; the method of evaluation of uncertainty of measurement by the statistical analysis of a series of measurements. An example would the standard deviation of a series of measurements taken by a laboratory technician.

Type "B" evaluation method; the method of evaluation of uncertainty of measurement by means other than the statistical analysis of a series of observations. An example would be the manufacturer's published specifications for an instrument.

Methods of determining uncertainty

Published specifications; as mentioned earlier published specifications are the most common source of uncertainty data used by commercial calibration laboratories. This method is the most appropriate for laboratories that take only "simple measurements". Simple measurements can be defined as any measurement that is within the common functional capabilities of an instrument. This method of determination is considered a Type "B" uncertainty.

Statistical methods; this method requires the taking of a series of measurements over a specified length of time. This method is the most robust and is appropriate for any laboratory that requires high confidence in their measurement uncertainty statements. This is a Type "A" uncertainty.

Distributions Associated with Measurement Uncertainty

The last piece of information that is needed to determine standard uncertainty is the distribution of the Type B uncertainty. There are four types of distribution:

- Normal distribution
- Rectangular distribution
- Triangular distribution
- U-shaped distribution

Normal distribution is usually associated with Type A uncertainty and has a divisor of one.

Rectangular distribution is used where there is an equal probability of a measurement occurring within the bound limits. This type of distribution is normally associated with manufacturer specifications. The GUM suggests assuming the rectangular distribution when the frequency distribution is not known. Rectangular distribution assumption will allow the laboratory to err on the conservative side. To convert a Type B specification with a rectangular distribution, divide the stated uncertainty by the square root of 3 to arrive at the standard uncertainty.

Triangular and U-shaped distributions will not be discussed in this paper, the assumption of these distribution classifications requires a sound understanding of statistical techniques that is beyond this paper's intent.

Uncertainty Budget

In the process of defining an uncertainty budget all of the most important contributors to uncertainty in the measurement must be considered. Once all of the contributors are defined they need to be normalized to standard uncertainty. The GUM provides the following correction factors for non-normal distributions.

Distribution	Divide by	Divisor
Rectangular	Square root of 3	1.7321
Triangular	Square root of 6	2.4495
U-Shaped	Square root of 2	1.4142

An example of this conversion; a manufacturer's accuracy specification for a multimeter at 100 volts is ±.5 volts. To convert this rectangular distribution to a standard uncertainty, divide .5 volts by the square root of 3 (1.7321) for a normalized uncertainty of .2886.

Combining Uncertainties

Once all contributors in an uncertainty budget have been converted to a standard uncertainty the standard uncertainties must then be converted to a unified unit of measure. The final step in any uncertainty budget is the combining of uncertainties. The process of combining uncertainties is called Root Sum Square or RSS. This means that each of the standard uncertainties is squared before adding all of the squared components together. The square root of the result is taken as the total combined standard uncertainty.

Uncertainty Formula:

$$U = \sqrt{S1^2 + S2^2 + S3^2 \ldots}$$

The expanded uncertainty is obtained by multiplying the resulting value U by a factor of two (*k=2*), approximating a 95% confidence level. This expanded uncertainty is shown in the uncertainty statement of a scope of accreditation.

Reporting

It is important that all of the steps used to arrive at a final uncertainty value be documented in a final report. This document can be as simple as a table outlining all of the considerations and the reasoning behind their consideration or removal. This report should be reviewed periodically by the laboratory management and updated as measurement equipment, personnel and or procedures change.

Creating a budget

1. List all possible contributors.
2. Decide on the Uncertainty Unit of measure.
3. Define the magnitude of the uncertainty contributors and their probability distributions.
4. Convert to standard uncertainty using the appropriate divisor and combine using RSS.
5. Document the basis for your estimates.
6. Review your uncertainty tables regularly.

Uncertainty Table for DC voltage standard

Uncertainty Contributors @ 1 VDC		Uncertainty (mV)	Probability Distribution	Type	Divisor	Standard Uncertainty
1	Repeatability	.0002	Normal	A	1	.0002
2	Long Term Drift	.00001	Normal	A	1	.00001
3	Specification	.0004	Rectangular	B	1.7321	.0002309
4	Thermal Stability	.00001	Rectangular	B	1.7321	.0000057
5	EMF, Cables	.00001	Rectangular	B	1.7321	.0000057
Combined Standard Uncertainty (RSS method)						.000305745
Expanded Uncertainty (*k*=*2*)						.000611489

Notes:
1 – Repeatability was determined using an HP 3458A multimeter and taking readings twice a day for one week.
2 – Long term drift was determined using data from two years of intermediate testing.
3 – Specification was taken from the manufacturer's specifications
4 – Thermal stability was taken from the manufacturer's specifications
5 – EMF, Cable specification was taken from the manufacturer's specifications

Table 1.0

Rounding

Rounding should only be performed once all calculations have been completed. As seen in Table 1.0 the expanded uncertainty is reflected to 9 places after the decimal. This number can now be rounded to a more practical value before being expressed on a scope of accreditation. The final expanded uncertainty should reflect the resolution of the laboratory's indicating device, plus one decimal place if desired. In the example above the rounded expression would be, ± .000612 mV DC.

Summary

There are many factors to take into consideration when expressing total and expanded uncertainties in DC and Low frequency metrology. This paper is intended to give the new technician some insight into the complexity of this discipline and start the process of critical thinking. This paper does not, by any means, cover the diversity and complexity of calibrations in today's metrology laboratory.

For those desiring more extensive information websites such as NCSLI.org or NIST.gov are good places to start.

References

ANSI/NCSL Z540-2-1997, U.S Guide to the expression of uncertainty in measurement. Boulder, CO: NCSL International (GUM)

Fluke Corporation. 1994. *Calibration Philosophy in Practice*, second edition. Everett, WA: Fluke Corporation.