Understanding and Expressing Measurement Uncertainties associated with Thermodynamic Metrology
(A primer)

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Introduction
In Thermodynamic Metrology all too often personnel are assigned to perform tasks with little or no experience. The importance of sound temperature metrology was of little interest or was not given the same respect as DC Low Frequency or RF microwave. Over the past 20 years temperature realization has become more important due to the effect it has on other metrology disciplines. There are many broad and some minute aspects to this discipline. This paper is intended as a primer to assist the new technician in understanding thermodynamic metrology, determination of measurement uncertainty and expression of uncertainty SI units.

Measurement of Temperature
A variety of instruments can be used to measure temperature; LIG (liquid in glass) thermometers, RTD's (resistive temperature detectors), thermocouples, SPRT's (standard platinum resistance thermometers) and a few others. Most of these with the exception of LIG's need an external device to read the signal from the sensing element.

Sources of uncertainty in this type of arrangement are the sensor, the indicating device, and in many cases, the connection method.

Heat Source
All heat sources introduce measurement errors as a result of their thermodynamic properties. This measurement error should be quantified to determine the heat sources contribution to the measurement uncertainty. All laboratories providing temperature calibration services maintain a variety of standards for realization of temperature depending upon the complexity or level of calibration being performed. Dry blocks, liquid baths and fluidized sand baths are all used in the comparison method of calibration, where a calibrated SPRT is compared to an unknown sensor. This method is typically used by secondary laboratories calibrating commercial temperature elements and is the most common type of thermodynamic calibration.

In the commercial laboratory, a less common method is the use of ITS-90 (International Temperature Scale – 1990) fixed point apparatus. These devices are based on fixed, known temperature points which are derived from physical constants of nature. Properly designed fixed point maintenance apparatus and the associated fixed point cell will maintain a known temperature over many hours and in some cases days. The most common fixed point is the triple point of water (TPW), which, when properly prepared can maintain 0.01°C with better than .0001°C of uncertainty.

There are many sources of uncertainty in a heat source; axial uniformity, radial uniformity, loading effect, temperature stability, stem conduction error, reference probe, and many others. We will discuss each of these briefly and how to arrive at a combined measurement uncertainty and an expanded uncertainty (k=2).

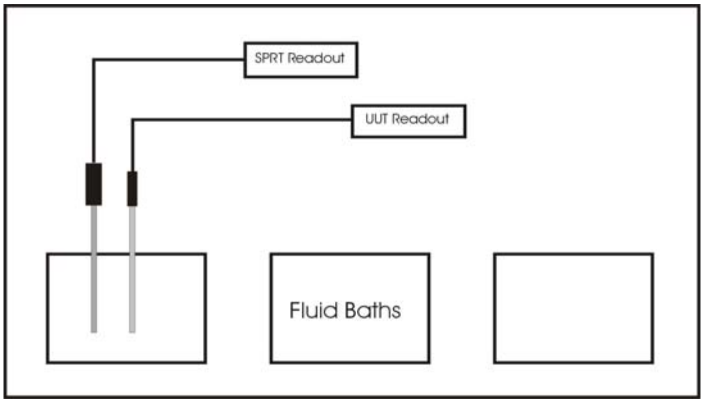

Figure 1. Diagram of comparison calibration

Uncertainties associated with comparison calibrations

The major sources of uncertainty are uniformity, stem conduction, loading, bath instability, reference thermometer accuracy, and unit under test accuracy.

Uniformity

The vertical gradient in a bath or dry well is termed “axial uniformity”. The surface of a bath or dry well is exposed to ambient environment and to a controlled temperature along a portion of the vertical length. This axial uniformity can be minimized be aligning the reference probe and UUT sensing centers.

The horizontal gradient in a bath or dry well is termed “radial uniformity”. The bath or dry well construction as well as the distance between sensors and the heating source can all lead to radial measurement uncertainty. Minimize this type of uncertainty by using a reference sensor of the same diameter as the UUT and placing the sensors as close together as possible.

Stem conduction error

Stem conduction is the parasitic loss of heat along the length of the thermometer stem. This affects both the reference and UUT. Following the recommended depth for minimum thermometer insertion (**20 X probe OD)** will minimize stem conduction error resulting in very little uncertainty.

Loading effect

The number of probes impacts the amount of heat drawn away from or into a bath or well. This effect can be minimized by allowing sufficient time for equalization and saturation of the work load before taking measurements.

Other Considerations

Drift

Control sensor and reference sensor drift will vary depending on the care and frequency of use of any bath or well. This drift can be determined with regular calibration and intermediate checks.

Control Sensor

When using a bath or well without an external reference the control sensor becomes another source of uncertainty. Minimize your uncertainty by using a reference sensor whenever possible.

Hysteresis

Hysteresis is the difference in indicated temperature dependant on the direction the bath or well approaches set point.

Uncertainty Budget

Any uncertainty budget should be an aggregate of the most important contributors from the previously mentioned sources of error. The most common method is to use the “Type B” approach. In this approach, the uncertainties published in the instrument manuals are used.
Some of the values needed for an uncertainty budget can be found on the calibration report for the instruments used. If uncertainties are required at points between those expressed on the calibration report, then rigorous calculating methods are needed to propagate uncertainties. The calibration lab can use the law of Propagation of Uncertainties as described in the GUM to achieve these uncertainties. However, a less complex method for establishing the uncertainties between calibration points would be a technique called

Linear Interpolation. In this technique we assume that the uncertainty changes linearly between the points of interest.

Combining uncertainties

We must first convert to standard uncertainty in order to evaluate the total uncertainty. At the same time we must ensure that all values are expressed in the same unit of measure, see "SI Units" below.

There are two types of uncertainties; uncertainties based on known or assumed probability distribution and those based on limits of error. When uncertainties are stated with a known distribution with a coverage factor of k=2, it means a normal, or Gaussian, distribution has been assumed. To convert this type of uncertainty to a standard uncertainty we simply need to divide by two. For other uncertainties types such as limits of error that were assigned without probability distributions, a rectangular distribution is assumed. To convert a rectangular distribution to a standard uncertainty, the value is divided by the square root of three. The individual standard uncertainties are then combined through a process called Root Sum Square or RSS. This means that each of the standard uncertainties is squared before adding all of the squared components together. The square root of the result is taken as the total combined standard uncertainty.

$$U = \sqrt{S1^2 + S2^2 + S3^2 \ldots}$$

The expanded uncertainty is obtained by multiplying the resulting value U by a factor of two (k=2). This expanded uncertainty is shown in the uncertainty statement of scope of accreditation.

Example

Illustrated below is a theoretical uncertainty table for a metrology bath with a standard thermometer as the readout. This example is at 0°C and does not take the UUT into consideration. Please remember this is theroretical, the math is sound the uncertainties are without basis.

Table 1

Uncertainty Contributors @ 0.0°C	**Specification (°C)**	**Probability Distribution**	**Uncertainty (°C)**
Stability	.002	Rectangular	.0011
Loading effect	.001	Rectangular	.0005
Radial uniformity	.002	Rectangular	.0011
Axial uniformity	.004	Rectangular	.0023
Reference probe calibration	.004	Rectangular	.0023
Reference probe drift, including hysteresis	.002	Rectangular	.0011
Reference probe stem conduction	.001	Rectangular	.0005
Thermometer readout accuracy	.001	Rectangular	.0005
Combined Standard Uncertainty (RSS method)			.003868
Expanded Uncertainty (k=2)			.007736

Rounding
Rounding should only be performed once all calculations have been performed. As seen in Table 1 the expanded uncertainty is reflected to 6 places after the decimal. This number can now be rounded to a more practical value before being expressed on a scope of accreditation or customer certificate of calibration. The final expanded uncertainty should reflect the resolution of the laboratories indicating device. In the example above the rounded expression would be, ±. 0.008°C for a three decimal place digital display.

SI Units
When starting the process of determining uncertainty, **always** convert non-standard units to SI units. The Kelvin and degree Celsius are the accepted units for expressing uncertainty. This conversion should be performed before any calculations are done. It should be noted that Kelvin is not expressed as °K, the Kelvin is an absolute value of temperature and should always be expressed as mK or K.
Note: 0°C = 273.16K.

Summary
There are many factors to take into consideration when expressing total and expanded uncertainties in thermodynamic metrology. This paper is intended to give the new technician some insight into the complexity of thermodynamic metrology. This paper does not, by any means, cover the diversity and complexity of Thermodynamic Calibrations in today's metrology laboratory. However, more extensive information can be gleaned from websites such as NCSLI.org or NIST.gov.